Large Scale Structure and Modified Gravity

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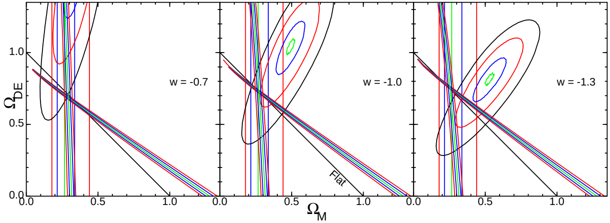
Outline

Motivation



- H_0 : vertical lines, BAO : almost vertical lines, CMB : inclined lines, SNe : ellipses [N.Wright 06]
- Current accelerating universe : GR with ordinary matter components fails to explain (Prof.Ko's talk for DM)
- $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \delta G_{\mu\nu} = 8\pi G T^{\rm fluid}_{\mu\nu} + 8\pi G T^{\rm DE}_{\mu\nu}$
- Other possibilities : Inhomogeneity models, String.

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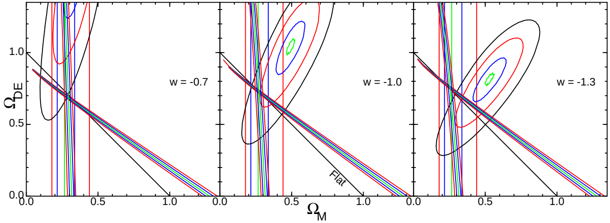

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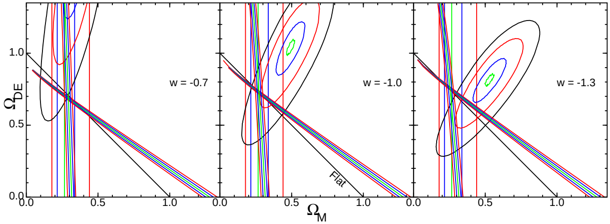

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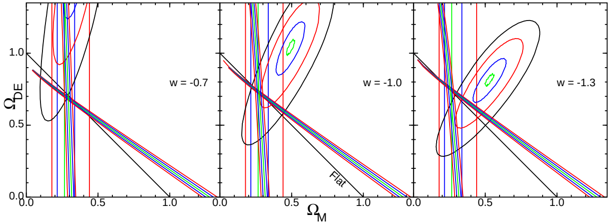

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Class of modified gravities I : BD

1. Brans-Dicke theory [61]
   - $S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi}\left(\phi R - \frac{\omega_{\rm BD}}{\phi}\nabla_\mu\phi\nabla^\mu\phi\right) + \mathcal{L}_{\rm fluid}\right]$
   - ϕ : BD field , $\omega_{\rm BD}$: BD parameter , $\mathcal{L}_{\rm fluid}$: m and(or) r
2. Limits on $\omega_{\rm BD}$:
   - BBN ($z \sim 10^{10}$) : > 50 [A. Serna 92]
   - CMB.LSS ($z \sim 10^3$) : $> 120(2\sigma)$ [V. Acquaviva 05, 07]
   - Solar System (Cassini spacecraft) : $> 40000(2\sigma)$ [B. Bertotti 03]

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Class of modified gravities II : DGP

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   - M : 5-dim Planck scale , $g^{(4)}$: induced metric on the brane
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Class of modified gravities III : $f(R)$

1. $f(R)$ gravities in Metric formalism [H.A. Buchdahl 70]
   - $S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} f\Big(R(g_{\mu\nu})\Big) + \mathcal{L}_{\rm fluid}(g_{\mu\nu}, \psi) \right]$
   - $f(R)$ is a general function of R
   - $\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta}(\partial_\mu g_{\nu\beta} + \partial_\nu g_{\mu\beta} - \partial_\beta g_{\mu\nu})$: Levi-Civita connection (assume $\nabla_\lambda g_{\mu\nu} = 0$)
2. $f(R)$ gravities in Palatini formalism [M. Ferraris 82, SL 07]
   - $S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} f\Big(\hat{R}(g_{\mu\nu}, \hat{\Gamma}^{\alpha}_{\mu\nu})\Big) + \mathcal{L}_{\rm fluid}(g_{\mu\nu}, \psi) \right]$
   - $g_{\mu\nu}$ and $\hat{\Gamma}^{\alpha}_{\mu\nu}$ independent
   - $\hat{g}_{\mu\nu} = F g_{\mu\nu}$, $\hat{R}_{\mu\nu} = \hat{\Gamma}^{\alpha}_{\mu\nu,\alpha} - \hat{\Gamma}^{\alpha}_{\mu\alpha,\nu} + \hat{\Gamma}^{\alpha}_{\alpha\beta}\hat{\Gamma}^{\beta}_{\mu\nu} - \hat{\Gamma}^{\alpha}_{\mu\beta}\hat{\Gamma}^{\beta}_{\alpha\nu}$

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Candidates of Dark Energy

1. Cosmological constant Λ [S.Weinberg 00]
   - $\frac{\rho_{\rm today}}{\rho_{\rm Planck}} = \left(\frac{10^{-3}}{10^{27}}\right)^4 \sim 10^{-120}$
2. Quintessence [D.Huterer 99, SL 05,06]
   - slowly rolling scalar field
   - early time : tracker solution ($\omega = \frac{1}{3}$), late time : fine-tune
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MG as DE I

- $H^2 - \delta H = \frac{8\pi G}{3}\rho_m$
- $H^2 \equiv \frac{8\pi G}{3}\left(\rho_m + \rho_{\rm DE}\right)$
- $\omega_{\rm DE} = -1 - \frac{1}{3}\frac{d\ln\delta H}{d\ln a}$

Table 1. Comparison of δH and $\omega_{\rm DE}$ in BD, DGP, metric formalism $f(R)$ theory, and Palatini formalism $f(R)$ models. $F(R) = \frac{\partial f(R)}{\partial R}$ and F_0 is the present value of $F(R)$.

	δH	$\omega_{\rm DE}$
BD	$\frac{\omega_{\rm BD}}{6}\frac{\dot\phi^2}{\phi^2} - H\frac{\dot\phi}{\phi}$	$-1 + \frac{\frac{\ddot\phi}{\phi} - H\frac{\dot\phi}{\phi} + \omega_{\rm BD}\frac{\dot\phi^2}{\phi^2}}{\frac{\omega_{\rm BD}}{2}\frac{\dot\phi^2}{\phi^2} - 3H\frac{\dot\phi}{\phi}}$
DGP	$\frac{H}{r_0}$	$-\frac{1}{1+\Omega_{\rm m}}$
Metric $f(R)$	$\frac{1}{3F_0}\left(\frac{1}{2}(FR - f) - 3H\dot F + 3H^2(F_0 - F)\right)$	$-1 + \frac{2\ddot F - 2H\dot F - 4\dot H(F_0 - F)}{FR - f - 6H\dot F + 6H^2(F_0 - F)}$
Palatini $f(R)$	$\frac{1}{3F_0}\left(\frac{1}{2}(FR - f) + \frac{3}{2}\ddot F + \frac{3}{2}H\dot F - \frac{3}{2}F\frac{\dot F^2}{F^2} + 3H^2(F_0 - F)\right)$	$-1 + \frac{2\ddot F - 2H\dot F - 3F\frac{\dot F^2}{F^2} - 4\dot H(F_0 - F)}{FR - f + 3\ddot F + 3H\dot F - 3F\frac{\dot F^2}{F^2} + 6H^2(F_0 - F)}$

MG as DE II

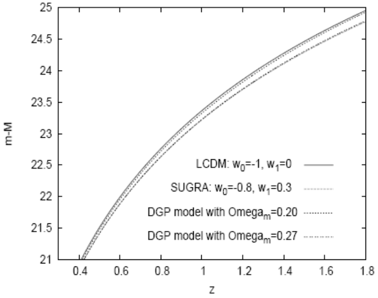
- Background and perturbations [SL 08]
- Discrepancies between models

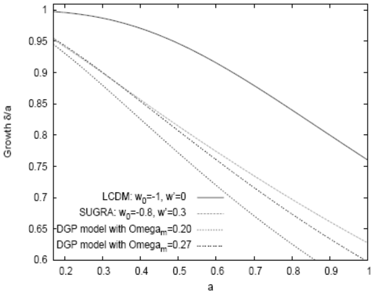
Table 2. Comparisons of $H(z)$ and evolution of the linear perturbation $\delta_{\rm m}$ with the effective gravitational constant $G_{\rm eff}$ in each model. In metric and Palatini formalism the perturbation calculations are held for subhorizon scale (i.e. $\frac{k}{a} > H$). Where $Q = -\frac{2F_{,R}}{F}\frac{k^2}{a^2}$ and $F'(T) = \frac{\partial F(T)}{\partial T}$.

	$\frac{H(z)}{H_0}$	$\delta_{\rm m}$	$G_{\rm eff}$
BD	$\sqrt{\frac{\delta H}{H_0^2} + \frac{\phi_0}{\phi}\Omega_{\rm m}^{(0)}(1+z)^3}$	$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\rm eff}\rho\delta = 0$	$\left(\frac{2\omega_{\rm BD}+4}{2\omega_{\rm BD}+3}\right)\frac{1}{\phi_0}$
DGP	$\frac{1}{2}\left(\frac{1}{r_0 H_0} + \sqrt{\left(\frac{1}{r_0 H_0}\right)^2 + 4\Omega_{\rm m}^{(0)}(1+z)^3}\right)$	$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\rm eff}\rho\delta = 0$	$G\left(1 + \frac{1}{3[1+2r_0 H\omega_{\rm DE}]}\right)$
Metric $f(R)$	$\sqrt{\frac{\delta H}{H_0^2} + \Omega_{\rm m}^{(0)}(1+z)^3}$	$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\rm eff}\rho\delta = 0$	$\frac{2(1-2Q)}{2-3Q}\frac{G}{F}$
Palatini $f(R)$	$\sqrt{\frac{\delta H}{H_0^2} + \Omega_{\rm m}^{(0)}(1+z)^3}$	$\ddot{\delta} + H\dot{\delta} - 4\pi G_{\rm eff}\rho\delta = 0$	$-\frac{1}{4\pi}\frac{F'}{2F+3F'\rho}\frac{k^2}{a^2}$

Background and perturbation

- Background evolution (Hubble parameter) [M.Ishak 06]
- Evolution of Perturbation (growth factor)





Metric and Fluid perturbations

1. line element : $ds^2 = -(1+2\Psi)dt^2 + (1-2\Phi)a^2(t)d\vec{x}^2$
   - Ψ : Newtonian potential (acceleration of particles)
   - Φ : spatial curvature perturbation
   - $\Psi = \Phi$ in GR without σ $\quad \Psi - \Phi = \frac{\delta F}{F}$ in $f(R)$ gravity
2. Energy momentum perturbations :
   - $\delta T^0_0 = -\delta\rho \quad , \quad \delta T^0_i = (\bar{\rho} + \bar{P})v_i$
   - $\delta(\vec{x}, t) = \frac{\rho(\vec{x},t) - \bar{\rho}(t)}{\bar{\rho}(t)}$: density fluctuation
   - $\theta \equiv \vec{\nabla} \cdot \vec{v}$: divergence of the peculiar velocity
3. Four quantities to be measured in the observations : to distinguish models

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Gravitational Lensing

- $\alpha_i = -\int \partial_i(\Psi + \Phi)ds$: deflection angle
- All lensing observables obtained by taking derivatives of α_i
- shear power spectrum for WL tomography
  $C_{\gamma_i\gamma_j}(l) = \int d\chi W_i(\chi) W_j(\chi) k^{-4} P_{\Psi+\Phi}\Big(k, \chi\Big)$
  where χ : a comoving distance, $W_i \sim \frac{\chi_i - \chi}{\chi_i}$: weight function
  $P_{\Psi+\Phi} = 9 k^{-4} H_0^4 \Omega^2 P_\delta a^{-2}$
- WL probes $\Psi + \Phi$ [E.Linder 08]
- SDSS : galaxy-galaxy lensing measured (Prof.Im's talk : BOSS).
- Detail will be given(?) : Prof.Chae's this afternoon talk

Velocity measurement

- $k^2\Psi = \frac{d(a\theta_g)}{dt}$: using galaxy satellite dynamics and rotation curve on sub-Mpc
- $k^2\Psi = \frac{d(aD_\theta)}{dt}\frac{\theta_g}{D_\theta}$: in the linear regime where $D_\theta \sim a\dot{D}$ is linear growth factor of θ
- multiple redshifts velocity measurements probes Ψ

Statistical measure of correlations in n_g

- $\delta_g = \frac{\delta n_g}{n_g} = b_1 \delta + \frac{b_2}{2} \delta^2$: galaxy density where b_1, b_2 bias parameters
- $C_g(l) = \int d\chi \frac{W_g^2(\chi)}{\chi^2} P_g(k, \chi)$: $3 - D$ galaxy power spectrum where W_g normalized redshift distribution of galaxies
- galaxy power spectrum probes δ

SNe, CMB

1. Distance Measurements
   - true distance − distance measured from the recession velocity
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   - : SNe, SZ effect, cluster peculiar velocity
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   - $C_{ISW}(l) = \int P_{g(\dot{\Psi}+\dot{\Phi})}(k,\chi) a^2 \chi^{-2} d\chi$

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